Posted Pricing versus Bargaining in Sequential Selling Process

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Bargaining: An Example

Consider a seller of a house negotiating with a buyer.

- The buyer has a valuation of $r$ for the house.
- The buyer, if walking away, obtains a value $v_B$.
- The seller, if keeping the house, obtains a value $v_S$.

Will they trade?

If so, what price $s^B$ would they settle at?

<table>
<thead>
<tr>
<th></th>
<th>Buyer</th>
<th>Seller</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Trade value</td>
<td>$r - s^B$</td>
<td>$s^B$</td>
<td>$r$</td>
</tr>
<tr>
<td>Disagreement point</td>
<td>$v_B$</td>
<td>$v_S$</td>
<td>$v_B + v_S$</td>
</tr>
<tr>
<td>Trade Surplus</td>
<td>$r - s^B - v_B$</td>
<td>$s^B - v_S$</td>
<td>$r - v_B - v_S$</td>
</tr>
</tbody>
</table>

Trade only if $r - v_B - v_S \geq 0$

Negotiated price depends on **bargaining power**
Rubinstein (1982)

- **Patience:** \( \delta_x = e^{-\mu_x \Delta t} \)

Sequence:
- Seller offers \( s_1 \)
- Buyer responds (accept, reject, withdraw)
- Seller responds (accept, reject, withdraw)
- If buyer rejects, he counteroffers \( s_2 \)
- If seller rejects, he counteroffers \( s_3 \)
- ... 

Fixed point:
\[
\omega^S = \max_{s_1} \{ s_1 - v_s \}
\text{s.t. } r - s_1 - v_B \geq \delta_B \omega^B
\]

\[
\omega^B = \max_{s_2} \{ r - s_2 - v_B \}
\text{s.t. } s_2 - v_s \geq \delta_s \omega^S
\]

When \( \Delta t \to 0 \):
\[
\frac{1 - \delta_B}{1 - \delta_B \delta_S} \to \frac{\mu_B}{\mu_S + \mu_B}
\]
Nash Bargaining (1950)

- Bargaining power of the buyer is $\theta \in [0,1]$

$$s^B = \arg\max_s \{(r - s - v_B)^\theta (s - v_s)^{1-\theta}\}$$

- Buyer’s surplus $= \theta (r - v_B - v_s)$
- Seller’s surplus $= (1 - \theta)(r - v_B - v_s)$

Equivalence of the two models: $\frac{\mu_B}{\mu_S + \mu_B} = 1 - \theta$
Bilateral Bargaining in OM Literature

- A single trade.

- Parallel trades.

- Sequential trades?
  - A firm’s trade with one partner affects his future trades with other partners
  - E.g., Revenue Management
The revenue management literature typically assumes that the seller dictates the price and the buyers are price takers.

- **Single product**: Gallego and van Ryzin (1994) and Bitran and Mondschein (1997)...
- **Multi-product**: Zhang and Cooper (2005), Maglaras and Meissner (2006), Popescu and Wu 2007, Ahn et al. (2007)...
- **Surveys**: Bitran and Caldentey (2003) and Elmaghraby and Keskinocak (2003)...
- **Strategic consumer**: Aviv and Pazgal (2008), Elmaghraby et al. (2008), Su (2007), Cho et al. (2009)...

What if the buyers want to negotiate? Should the seller be open to bargaining?
Posted Price versus Bargaining (Static)

- **Bester 1993**
  Fixed versus negotiated price based on product quality

- **Adachi 1999, Desai and Purohit 2004**
  Fixed versus negotiated price based on location

- **Ellingsen and Rosen (2003)**
  Fixed versus negotiated wage based on productivity

.....
A Sequential Trading Problem

Two closely related papers:

- Bhandari and Secomandi (2011, OR)
1. The seller posts a price $s^l$, but is open to bargaining with a reserved price $c$.
2. An arrival of buyer occurs in each period with probability $\lambda$.
3. A buyer has a random valuation $R$, observed upon arrival.
4. A fraction of $\rho$ buyers are up for bargaining.
5. No outside options for the seller and the buyers

Negotiated price with buyer of valuation $r$ is

$$s^B(r) = \arg\max_s \{(r - s)^\theta (s - c)^{1-\theta}\} = (1 - \theta)r + \theta c$$

DP equation with $n$ units of product in period $t$

$$V(t, n) = \max_{s^l, c} \{\lambda \rho (E[s^B(r) \wedge s^l] + V(t + 1, n - 1))$$
$$+ \lambda(1 - \rho)F(s^l)[s^l + V(t + 1, n - 1)]$$
$$+ [1 - \lambda \rho - \lambda(1 - \rho)F(s^l)]V(t + 1, n)$$}

Trade: negotiators
Trade: price-takers
No trade
1. The selling scheme is determined exogenously.
2. An arriving buyer assumes that the seller’s valuation of the product is \( V_1 \sim \text{Uniform}[0,1] \) and the seller assumes that an arriving buyer’s valuation of the product is \( V_2 \sim \text{Uniform}[0,1] \).
3. An arrival of buyer occurs in each period with probability \( \lambda \).
4. The trading contract specifies a probability \( p(v_1, v_2) \) of trading and a price \( x(v_1, v_2) \) paid from the buyer to the seller.
5. Infinite horizon and discount factor \( \beta \in [0,1) \).
6. No outside options for the seller and the buyers.

**Applied Myerson’s and Chatterjee and Samuelson’s approach (direct mechanism) for incentive compatible mechanism.**

**DP equation with \( n \) units of product**

\[
V(n) = (1 - \lambda)\beta V(n) + \lambda \max_{v_1} \left\{ E \left[ x(v_1, V_2) + \beta V(n - 1)I_{\{\text{deal}\}} + \beta V(n)I_{\{\text{no deal}\}} \right] \right\}
\]
Unresolved Issues

- **Complete information:** Modeling of the dynamics between the trades?

- **Incomplete information:** Mechanism? Information specification?
  - Ayvaz-Cavdaroglu, Kachani, Maglaras (2014): min max regret.
The Problem

- The seller firm has a fixed stock of product to be sold in a given planning horizon
- A stream of buyers with random valuation $R \sim F(\cdot)$
Research Questions

- Is pricing better or bargaining better?
  - When there is a large stock to sell
  - When the selling season is short
  - Characteristics of buyer population
Our Approach

- Single Trade
  - Bargaining only
  - Pricing only
- Dynamic Problem with Sequential Trade
  - Functional property
  - Policy structure
- Model Variations
A buyer arrives and, if bargaining, reveals valuation $R = r$

Buyer purchases or quits

Seller decides

[P] Pricing only ($s^P$)

[B] Bargaining only ($s^B$)
Potential Buyer Population

The seller should only consider a buyer whose valuation is above his disagreement point. The trade surplus is:

\[ R(v) = \{ R - v | R > v \} \]

\[ \rightarrow \text{Bargaining: } \Psi^B(v) = (1 - \theta)E[R(v)] + v \]

Obtains \((1 - \theta)\) portion of expected trade surplus

\[ \rightarrow \text{Pricing: } \Psi^P(v) = \max_{s^P \geq v} [(s^P - v)\bar{F}_R(s^P) + v] \]

\[ = g(v)E[R(v)] + v \]

What is \(g(v)\)?
Mean-Scaled Residual Life Distribution

- Define \((the \ portion \ of \ expected \ surplus \ earned \ by \ pricing)\)
  \[g(v) = \max_{\tau \geq 0} \{\tau F_R(v)(\tau E[R(v)])\}\]

- Monotone property of \(g(v)\) implies the comparison between pricing and bargaining

- When is \(g(v)\) in variant, increasing or decreasing?

A random variable \(X_1\) is said to be stochastically larger than another random variable \(X_2\) in \textit{scaled pricing order}, written \(X_1 >_{scpr} X_2\), if

\[\max_{x \geq 0} \{x F_{X_1}(xE[X_1])\} = \max_{x \geq 0} \{xF_{X_2}(xE[X_2])\}\]
Examples: Invariant Distribution Family

\[ \bar{F}_{R(v)}(\tau E[R(v)]) = \bar{F}(\tau E[R]) \]

Uniform: \( F(r) = r, r \in [0,1] \)

Exponential: \( F(r) = e^{-0.5r}, r \in [0,1] \)
Invariant Distribution Family

\[
\max_{\tau \geq 0} \tau \overline{F}_{R(v)}(\tau E[R(v)]) = \max_{\tau \geq 0} \tau \overline{F}(\tau E[R]) \text{ if and only if}
\]

(i) \(\overline{F}(r) = \left(\frac{r-r_{\text{min}}}{r-r_{\text{max}}}\right)^{\alpha}, \ r \in [r_{\text{min}}, r_{\text{max}}], \alpha > 0, \) or

(ii) \(\overline{F}(r) = e^{-\alpha r}, \ r \in [0, \infty), \alpha > 0.\)

e.g., \(\overline{F}(r) = \left(1 - \frac{r}{r_{\text{max}} + \alpha}\right)^{\alpha}, \ r \in [0, 1 + \alpha)\)

Note \(E[R] = 1\)
Example: Monotone Distribution Family

Suppose $R$ belongs to the Kumuraswamy (MaxMin) family, i.e., $F(r) = (1 - r^\alpha)^\beta$, $r \in [0,1]$ for some $\alpha > 0$ and $\beta > 0$. The sequence $R(v) = \{R - v|R > v\}$ is increasing (invariant, decreasing) in the scaled pricing order if $\alpha < (=, >)1$.

$$F(r) = 1 - r^\alpha$$

*Increasing scaled pricing order ($\alpha = 0.7$)*

*Constant scaled pricing order ($\alpha = 1$)*

*Decreasing scaled pricing order ($\alpha = 1.5$)*
Understanding SCPR Order

Which buyer population is more attractive to the seller under pricing?

\[ \overline{F}^1(r) = 1 - r, \quad r \in [0,1) \quad \text{vs.} \quad \overline{F}^2(r) = 1 - \frac{r}{2}, \quad r \in [0,2). \]

\[ \overline{F}^1(r) = e^{-2r}, \quad r \in [0,\infty) \quad \text{vs.} \quad \overline{F}^2(r) = e^{-r}, \quad r \in [0,\infty). \]

\[ \overline{F}^1(r) = 1 - \frac{r}{2}, \quad r \in [0,2) \quad \text{vs.} \quad \overline{F}^2(r) = e^{-r}, \quad r \in [0,\infty). \]
Characterization

\[ g(v) = \max_{\tau \geq 0} \{ \tau \overline{F}_R(v)(\tau E[R(v)]) \} \]

- Suppose \( R \) has an increasing length-biased hazard rate, i.e., \( rh_R(r) \) is increasing.
  \[ X \in ISCP(R(DSCP)) \iff \overline{F}_R(v)(E[R(v)]) \text{ is decreasing (increasing) in } v. \]

- \( \overline{F}_R(v)(E[R(v)]) \) is decreasing in \( v \) \( \Rightarrow \) \( X \in ISCP(R) \).

- \( 1/h_R(r) \) is concave (convex) \( \Rightarrow \) \( \overline{F}_R(v)(E[R(v)]) \) is decreasing (increasing) in \( v. \)
Application to Our Problem

Bargaining: $\Psi^B(v) - v = E[R(v)](1 - \theta)$

Pricing: $\Psi^P(v) - v = E[R(v)]\max_{\tau \geq 0}\{\tau \bar{F}_{R(v)}(\tau E[R(v)])\}$

$\Rightarrow$ Pricing gets better (worse) if ISGPR (DSCPR) as $v$ increases.
Preference on Pricing Mechanism

If \( R(v) = \{ R - v \mid R > v \} \) is

- **invariant** in scaled pricing order, the seller chooses pricing (bargaining) if \( \theta < (>\theta) \) for some \( \theta \);

- **increasing** in scaled price ordering, then there exist \( v^i \) such that the seller chooses pricing (bargaining) \( v > (<) v^i \);

- **decreasing** in scaled price ordering, then there exist \( v^d \) such that the seller chooses pricing (bargaining) \( v < (>\) v^d \).
Dynamic Model: What is Disagreement Point?

- The seller’s optimal value with \( n \) units of stock at the beginning of period \( t \) is \( V(t, n) \)

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</tr>
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<tbody>
<tr>
<td>Trade</td>
<td>( r - s^B )</td>
<td>( s^B + V(t + 1, n - 1) )</td>
</tr>
<tr>
<td>No Trade</td>
<td>0</td>
<td>( V(t + 1, n) )</td>
</tr>
<tr>
<td>Surplus</td>
<td>( r - s^B )</td>
<td>( s^B - [V(t + 1, n) - V(t + 1, n - 1)] )</td>
</tr>
</tbody>
</table>

- The seller’s *disagreement point* for the particular unit
  \[ v = V(t + 1, n) - V(t + 1, n - 1) \]
The DP Formulation

\[ V(t, n) = \lambda \Psi[V(t + 1, n) - V(t + 1, n - 1)] + \lambda V(t + 1, n - 1) + (1 - \lambda)V(t + 1, n) \]

Recall, \( \Psi(v) = \max\{\Psi^P(v), \Psi^B(v)\} \)

- **Properties:** If \( \Psi(v) \) is increasing in \( v \) with slope \( \leq 1 \)
  - \( V(t, n) \) is increasing and concave in \( n \)
  - \( V(t, n) \) is decreasing and concave in \( t \)
  - \( V(t, n + 1) - V(t, n) \) is decreasing in \( t \)

- **Disagreement point is higher when**
  - Less stock on hand
  - More periods to go
Dynamic Pricing Strategy

If \( R(v) = \{R - v| R > v\} \) is

- **invariant** in scaled pricing order, the seller chooses pricing (bargaining) if \( \theta < (>)\hat{\theta} \) for some \( \hat{\theta} \);

- **increasing** in scaled price ordering, then there exist \( t^i(n) \) such that the seller chooses pricing (bargaining) \( t < (>)t^i(n) \);

- **decreasing** in scaled price ordering, then there exist \( t^d(n) \) such that the seller chooses bargain (pricing) \( t < (>)t^d(n) \).
Example

\[ F(r) = 1 - r^\alpha \]

*Increasing* scaled price ordering

\[ \alpha = 0.5, \theta = 0.55, \lambda = 1 \]

*Decreasing* scaled price ordering

\[ \alpha = 1.5, \theta = 0.47, \lambda = 1 \]
Variation 1: Commission-Based Bargaining Cost

- Seller hires an agent to bargain with the buyers
  - Pays a faction $\gamma$ of her gain to the agent for each successful deal

- Seller’s bargaining profit is
  $$\psi^{BC}(v) = (1 - \gamma)(1 - \theta) \int_{v}^{\bar{r}} \bar{F}_R(r) dr + v$$

- The structure of the optimal policy remains unchanged
Variation 2: Transaction-Based Bargaining Cost

\[ F(r) = 1 - r^\alpha \]

*Increasing scaled price ordering*
\[ \alpha = 0.5, \theta = 0.5, \lambda = 1, c_s = 0.01 \]

*Decreasing scaled price ordering*
\[ \alpha = 1.5, \theta = 0.45, \lambda = 1, c_s = 0.01 \]
Variation 3: Mix of Bargainer and Nonbargainer

- **Invariant**: always pricing or always bargaining

\[
\overline{F}(r) = 1 - r^\alpha
\]

*Increasing* scaled price ordering

\[
\alpha = 0.5, \theta = 0.55, \lambda = 1
\]

*Decreasing* scaled price ordering

\[
\alpha = 1.5, \theta = 0.46, \lambda = 1
\]
Other Variations

As long as $\Psi(v)$ is increasing in $v$ with slope $\leq 1$, single crossing of $\Psi^P(v)$ and $\Psi^B(v)$ implies threshold switching policy

- Long-term vs. short-term pricing decision
- Heterogeneous buyer bargaining power
- Non-stationary valuation distribution
- Non-stationary arrival rate
- Bulk purchase by one buyer
- …
Comparison to Previous Results

- **Kuo, Ahn, Aydin (2011, OR)**
  - Switching from pricing to bargaining over time
  - *Wang (1995): Bargaining dominates pricing for* \( T = \infty \)

- **Bhandari and Secomandi (2011, OR)**
  - Pricing always dominates bargaining
Conclusion

- It is important to carefully model the **disagreement point** in sequential bargaining process
  - Single-trade value depends critically on disagreement points
  - It hinges on the dynamics of strategy choice

- **Pricing vs. bargaining**
  - Highly sensitive to the heterogeneity of buyer’s **valuation**
  - Observation from a specific valuation distribution may not be generalized.
Thank you!

Questions?